On the Occasion of Prof. I. Dimovski's 70^{th} Anniversary

Professor D. Sc. **Ivan DIMOVSKI**, Institute of Mathematics and Informatics - Bulgarian Academy of Sciences (B.A.S.), Corresponding Member of B.A.S. and a **Member of Editorial Board of our "FCAA" Journal**, born on July 7, 1934, had in year 2004 his 70'th jubilee.

Prof. Dimovski started his successful mathematical carrier yet as a pupil in the secondary school in the town of Troyan, near to his born village. As a winner in the first national mathematical olympiad he received a prize, and as the year was 1950, this prize was the books collection of Lenine's works - nowadays an antique, a fact that he remembers now with a sense of humor. Next he had graduated the Mathematics Dept. at Sofia University, and another reason to be proud, was his active participation in the seminar of Prof. Jaroslav Tagamlitski on topics of real analysis, where many other students took part, later becoming known scholars. There is a period in his studies and research interests dedicated also to Mechanics. His carrier as a teacher and as assistant professor started in the town of Rousse, then continued in Sofia in the Institute of Mathematics of Bulgarian Academy Sciences, till nowadays - from a young mathematician (1959); through full professor (1982); a long-year chief of "Complex Analysis" section (1986 - 2004); lecturer in many Bulgarian universities on large variety of courses (calculus, history of mathematics, operational calculus, potential theory, theory of elasticity and continuum mechanics); as a corresponding member of Academy (1997); President of Scientific Council on "Applied Mathematics and Mechanics" (2005-).

Prof. Dimovski has more than 80 scientific publications in Bulgarian and international journals, including a monograph by Kluwer, *"Convolutional Calculi"*, referred to by other authors more than 500 times. Many papers, theses and monographs of his Bulgarian collaborators and of foreign authors have been inspired by his ideas and results and use them essentially, even some of them containing his name in the titles or in names of notions.

He has been an invited speaker at many international conferences and visiting professor in foreign universities, in: Russia, Germany, Yugoslavia, Poland, Spain, Venezuela, Kuwait, etc; member of the Editorial Boards of the international journals *"Integral Transforms and Special Functions"* and *"Fractional Calculus and Applied Analysis"*; reviewer for other international journals and publishers; organizer of a series of international mathematical conferences.

Among his contributions is the creation of a Bulgarian mathematical school (8 Ph. D. students, many M.Sc. students and research collaborators), as well as a group of foreign collaborators and followers, in the field of operational calculus and integral transforms.

In his young years, Prof. Dimovski translated more than 50 famous mathematical books from foreign languages (English, German, Russian and French) for the audience of Bulgarian mathematicians.

Another trend of his activities is the field of "school mathematics" - as a lecturer for pupils prepared for mathematical olympiads, and as author of many classroom books on elementary mathematics.

For the scientific achievements and contributions to Bulgarian science, on the occasion of his 70th jubilee, Prof. Ivan Dimovski was awarded the honor medal "Marin Drinov" of Bulgarian Academy of Sciences.

It is a pleasure to express my personal relations and gratitude to Prof. Dimovski, who had been advisor of both my Mr.Sc. thesis (1975) and Ph.D. thesis (1986), and a colleague for more than 30 years. All my research had been influenced by this starting point and by our further collaboration. He had introduced me to the field of integral transforms, proposing me as a task of the Mr.Sc. thesis, to continue his studies on the Obrechkoff integral transform. The relationship of its kernel function to Meijer's G-function generated my further interest in special functions, and the hyper-Bessel operators and their fractional powers, expressed in terms of integral operators involving G-functions gave rise to the theory of the generalized operators of integration and differentiation of fractional (multi-)order, developed in my monograph "Generalized Fractional Calculus and Applications".

On behalf of myself, my colleagues from "Complex Analysis" Section in Institute and the Editorial Board of the "FCAA" Journal, I like to wish him a health, fruitful continuation of his scientific activities and enthusiasm in imposing and popularization of new ideas in Mathematics.

SHORT ANNOTATION OF PROF. DIMOVSKI's CONTRIBUTIONS

The key role in Prof. Dimovski's studies, from his very first publication [11] to the recent ones, as he confesses himself, is played by the term "*convolution of a linear operator*". A new notion in mathematics usually deserves a long-standing role, only if it helps to solve problems that can be formulated without its use, but their solution requires an essential use of it. Such a notion happens to be the one introduced by Dimovski, the base of his "*convolutional approach*". By means of this approach, he has built new operational calculi for local and nonlocal boundary value problems, extending the area of applicability of the multipliers theory and relating it to the theory of commuting linear operators. Based on this convolutional approach, a new variant of the Duhamel principle has been developed, for a large variety of important nonlocal BVPs for equations of mathematical physics.

The classical operational calculus of Jan Mikusinski is based on the well known convolution of Duhamel,

$$f * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau. \tag{1}$$

Yet in his first scientific paper [11] of 1962, Dimovski proved that putting on the base of the operational calculus any other continuous convolution of the integration operator, leads to another operational calculus, isomorphic to Mikusinski's one. The idea to generalize the direct algebraic approach of Mikusinski for building operational calculi for other operators different from integration, has encountered both conceptual and technical problems, yet in the first attempts by some Russian, German and Hungarian mathematicians. Dimovski's notion "convolution of a linear operator" opened the way to such generalizations, allowing to speak about "operational calculi" (in plural form). Its origin is hidden yet in [11] and formulated for some particular operators and spaces in [42], with the most important examples for convolutions proposed in [17],[18],[31], to justify its general character. Since in each particular case of an operator, one needs to solve the problem of constructing a convolution in explicit form, the happy hint for Dimovski has been to start with (1966-1974) a very general class of operators of Bessel type or arbitrary order $m \geq 2$ (nowadays called "hyper-Bessel operators") and then to continue with the general linear differential operators of first and second order. The whole theory with the known applications (by then) can be found in his monograph [67]: Ivan Dimovski, ***Convolutional Calculus***, Kluwer Academic Publishers, Dodrecht - Boston - London, 1990 (its first edition being by Printing House of Bulg. Acad. Sci. in 1982).

Dimovski's basic definition is the following: *A bilinear, commutative and associative operation* $* : X \times X \mapsto X$ *is called a convolution of the linear operator* $L : X \mapsto X$*, mapping a given linear space* X *into itself, when* $L(f * g) = (Lf) * g$ *for all* $f, g \in X$.

To find a convolution of a given linear operator in explicit form, as a rule, is a difficult and nontrivial task. However, once found, a convolution operation allows easily to build an operational calculus for the corresponding operator, and thus to solve the basic spectral problems, related to it: finding various spectral functions of this operator, and also of its commutant. In this respect, the paper [8] is important, including a fact unknown by then: if such an operator has a cyclic element, then the rings of the multipliers of a nontrivial convolution of it, and of the operators commuting with it, coincide! This allowed Dimovski to find an elegant explicit characterization of the linear operators $M : C[0,1] \mapsto C[0,1]$, commuting with the classical integration operator $Lf(t) = \int\limits_0^t f(\tau)d\tau$. In the monograph [67] it is proven that for the commutation of M and l it is necessary and sufficient that M has an integral representation of the form $Mf(t) = \frac{d}{dt}\int\limits_0^t f(t-\tau)\alpha(\tau)d\tau$, where $\alpha(\tau)$ is a function simultaneously continuous and with bounded variation. This seems to be a result of wide mathematical importance.

The main approach in finding new convolutions, using some already known basic ones, happened to be the "*similarity method*", called also "*method of transmutations*". For the first time, Dimovski formulated this idea in [5] and with good justifying examples, in [9]. The method of similarity is based on the following simple fact: If $T : X \mapsto \widehat{X}$ and $\widehat{*}$ is a convolution of the linear operator $\widehat{L} : \widehat{X} \mapsto \widehat{X}$, then the operation $f * g := T^{-1}[(Tf)\widehat{*}(Tg)]$ is a convolution for the linear operator $L = T^{-1}\widehat{L}T$, mapping X into X. Usually, T is called a transmutation or similarity operator, and L and $\widehat{L}$ are similar operators.

Dimovski used essentially this approach, for the first time, in the case of Bessel-type operators, [56],[45],[46],[57]. The corresponding similarity operator T found by him, is a generalization of both classical transformations of Poisson and Sonine (nowadays, I use to call it a "Poisson-Sonine-Dimovski" transformation). Next, by means of the similarity operators of Delsarte-Povzner, he proved the possibility for building operational calculi not only for initial value problems for the general 2nd order differential operator ([32]) but also for a wide class of nonlocal boundary value problems, including as a very special case the famous Sturm-Liouville problem ([23],[24],[40],[41]).

From a point of view of wide-range mathematics, Dimovski himself considers as most valuable the contributions related to the spectral theory of the classical (1st order) differential operator. In the 30's of the 20th century, the French mathematician Jean Delsarte (the founder of "Bourbaki" group) made several unsuccessful attempts to find a convolution, related to the most general spectral problem for the differentiation operator. His only achievement then was the generalization of the classical Taylor formula for this operator. The general spectral problem for the differentiation operator (in a corresponding space) consists in studying the resolvent operator L_λ, the result of its action, $y = L_\lambda f$ being considered as a solution of the nonlocal boundary value problem (BVP) $y' - \lambda y = f$, $\Phi(y) = 0$, where Φ is an arbitrary nonzero linear functional.

Dimovski solved the problem of Delsarte in 1974 (papers [13],[14]) when he found and proved that the operation

$$(f * g)(t) = \Phi_\tau \left\{ \int_\tau^t f(t + \tau - \sigma) g(\sigma) d\sigma \right\} \tag{2}$$

is a convolution of the resolvent L_λ, and built the respective operational calculus ([15]). The same convolution was rediscovered independently, by the German mathematician Lothar Berg (member of German Academy of Sciences) in 1976, who made reference to Dimovski's paper of 1974, thus acknowledging his priority. This convolution (called now as Dimovski-Berg convolution) allowed to obtain a complete solution of the problem for multipliers of the Leontiev extensions in exponentials in the complex domain (papers [16],[27]).

Interesting analogues of the differentiation operator are the operator for backward shift translation $\Delta f(t) = (f(t) - g(t))/t$ (called also the Pommiez operator) and the finite-difference operator in the space of sequences. It was confirmed that Dimovski's general scheme works successfully also in these two cases (see [24],[21],[22]). The experts acknowledged strongly also the results on finding commutant of the Gelfond-Leontiev integration operator, from our joint papers [28],[29], referred to in the book of M.K. Fage and N.I. Nagnibida, "Problem of Equivalency of Ordinary Differential Operators", Nauka, Novosibirsk, 1987 (In Russian).

From the point of view of applied mathematics, the most important Dimovski's results concern the convolutions for nonlocal BVP for 2nd order linear differential operators, since these operators play basic role in the problems of mathematical physics. In the mathematical literature, there has been a lack of a well-developed theory of nonlocal BV problems. Each of the few authors studying such BVPs, has considered some very special cases, without any general view on these kind of problems. As to the local BVPs of mathematical physics, the most popular method for their solving is the Fourier method. Unfortunately, this method hardly allows a computer realization. Concerning the widely known difference methods, the experts confess that "taking into account the subsequent principle of calculation of the contemporary computers, such an approach requires a.... expense of time"(e.g. V.Z. Alad'ev, M.L. Shishakov, "Automated Working Space of a Mathematician", Moscow, 2000 (in Russian), p. 644). Anyway, by recently it seems nobody has tried to solve, by means of common used PCs, serious local and nonlocal BV problems related to equations of mathematical physics. Dimovski has proved, at least at principle, the possibility of using the *"convolutional method"* for such a task. The essence of his approach consists in combining the Fourier method with the Duhamel principle, papers [58]-[65]. The weakness of the Fourier method is in the necessity to use expansions of the boundary functions in series of eigenfunctions. This procedure requires calculating of number of integrals (from dozens to hundreds) and afterwards, summing the series obtained as solution in many points. In his papers [31],[41],[33],[67] Dimovski proposes convolutions for BV problems for Sturm-Liouville operators with one local, and another - in general - nonlocal boundary value conditions, and thus opening the way to extend the Duhamel principle from a time-variable to space-variables in linear problems of mathematical physics. Generally speaking, the Duhamel principle consists in finding all solutions of a BVP by means of one particular solution of same problem. Such a particular solution is defined in terms of simple boundary functions and does not require numerical computation of definite integrals. Combining the Fourier method with the Duhamel principle allows to avoid two hard stages, from computing point of view, in the realization of Fourier method: expansion of the boundary functions in series of eigen- and associated functions, and the summation in many points of the obtained solution which is a rule, a slow convergent series. The numerical experiments ([76]) show high efficiency of such a convolutional approach. A natural question arises about "If the combination of these two methods is so efficient both in theoretical and computational aspects, why nobody had the hint before, to use it?" Possibly, the answer is that nobody had expected the existence of a simple explicit expression for the corresponding convolutions of [31] ...

Another application of Dimovski's convolutions for nonlocal BVPs for 2nd order linear ordinary differential operators, is the possibility to generalize the notion of finite integral transformation for each of the corresponding BV problems ([40],[41]). This solves automatically also the problem of obtaining of explicit convolutions for these finite integral transforms. As a special case, it is obtained a solution of the problem posed in 1972 by Churchill, to find convolutions of the finite Sturm-Liouville transformations (see Dimovski's monograph [67]). As a by-side product of the convolutions related to the general Bessel-type operators (paper [43]) it is found an explicit convolution of the classical Meijer transform (paper [37]). Another well appreciated Dimovski's result is the explicit convolution of the discrete Hermite convolution ([39]). A whole chapter is dedicated to this result in the book *"Integral Transforms and Their Applications"* by L. Debnath, one of the pioneers in searching for such a convolution.

From the point of view of Bulgarian mathematics and its traditions, the most important Dimovski's contribution is the identification, studying and giving an international popularity to the so-called *"Obrechoff integral transform"*, widely popular nowadays generalization of the Laplace transform ([14],[15],[37],[75], etc). For his achievements on the subject, Dimovski was awarded with the "Nikola Obrechkoff" Prize of Academy, 1979. The Bulgarian mathematician Nikola Obrechkoff (1896-1963, see http://profot.fmi.uni-sofia.bg/about-nikola-obreshkoff), himself never claimed for an authorship of a new integral transform but only for a formula for integral representation of functions on the real half-axis, as an extension of a result of S. Bernstein. In 60's-70's Dimovski studied the so-called general Bessel type differential operators, i.e. the singular differential operators of arbitrary order $m \geq 2$ naturally extending the 2nd order Bessel operator,

$$B = x^{\alpha_0}\frac{d}{dx}x^{\alpha_1}\frac{d}{dx}x^{\alpha_2}\dots\frac{d}{dx}x^{\alpha_m} = x^{-\beta}\left(x\frac{d}{dx}+\beta\gamma_1\right)\dots\left(x\frac{d}{dx}+\beta\gamma_m\right)$$

$$= x^{-\beta}\left[x^m\frac{d^m}{dx^m}+a_1x^{m-1}\frac{d^{m-1}}{dx^{m-1}}+\dots+a_{m-1}x\frac{d}{dx}+a_m\right], \quad 0<x<\infty, \tag{3}$$

nowadays known as *"hyper-Bessel differential operators"*. Developing operational calculi for the corresponding integral operators L, initial right inverse to B ($BL = I$), he had occurred the idea that the Obrechkoff integral transform of 1958 (a slight modification of it) can be used successfully as a transform basis, in the same way as the Laplace transform is used in the classical operational calculus for the usual differentiation/ integration operators. Later on, the studies on the Obrechkoff transform and hyper-Bessel operators have been prolonged in some joint papers of Dimovski and Kiryakova, and extended to a theory of generalized fractional calculus and related to important classes of special functions in the monograph: V. Kiryakova, "*Generalized Fractional Calculus and Applications*", Longman - J. Wiley, Harlow - N. York, 1994. The keys of these further developments were given by: considering the fractional powers of the hyper-Bessel operators as the simplest "generalized operators of fractional integration and differentiation"; and by the role of some Meijer G-functions as kernel-functions of the Obrechkoff transform and of the hyper-Bessel integral operator, as well as solutions of classes of hyper-Bessel differential equations.

Another important result, related to the Bessel type operators of arbitrary order, is the wide generalization of he classical transmutation operators of Sonine and Poisson (papers [56],[54],[55],[45],[52]), by means of which the equivalency of every two Bessel type operators of one and same order is proven. For the applications, the generalized Sonine operator is important, since it transforms an arbitrary Bessel-type differential operator of order m into the m-tuple differentiation $(d/dx)^m$. It is worth mentioning that in each particular case, the corresponding Dimovski's formula gives the best possible result. This is a solution of the Delsarte problem, posed by him in a manuscript (about 80 p.) published only posthumously in 1970 in the 2nd volume of his collected papers.

There exists a field of mathematics, which would look like quite different today if there were not the contributions of Dimovski, the "*Operational Calculus*", nowadays extended to so-called "*Convolutional Calculus*". In "Mathematical Reviews" it is classified as a section A44. It leads its origin from the studies of O. Heaviside in the end of 19th century, and remained without a strong mathematical formulation by the 50's of 20th century, when the Polish mathematician Jan Mikusinski proposed his direct algebraical approach based on the classical Duhamel convolution (1), thus justifying the Heaviside calculus. Predecessors of Mikusinski were Volterra and Pérèz (1924, 1943) to whom belonged the idea of using convolutional fractions for the same purpose. In 1957 the Russian mathematician V.A. Ditkin gave an example of operational calculus, different from Mikusinski's one. Namely, while Mikusinski's calculus is concerned with the Cauchy problem for the differentiation operator d/dx, Ditkin's calculus concerns the same problem but for the simplest operator of Bessel type, $(d/dx)x(d/dx)$. New examples of operational calculi of Bessel type of rather particular forms appeared in the 60's, by different authors. In the papers [42],[43],[44],[45],[19] Dimovski established the applicability of Mikusinski's approach to the most general Bessel-type operator (3). In [44] and [56] he proved, for the first time, that the operational calculi for all Bessel-type operators in the Mukusinski scheme, are isomorphic. Especially, they are isomorphic to the Mikusinski operational calculus, since the classical differential operator is also a Bessel-type operator. In a paper [32] joint with N. Bozhinov, Dimovski proved that the operational calculi for initial value problems for the general 2nd order linear differential operator are also isomorphic to Mikusinski's calculus.

Conceptually new are Dimovski's contributions for building operational calculi for boundary value problems (BVPs) and especially, for nonlocal BVPs for linear differential operators of 1st and 2nd order. In contrary to the "algebraic analysis" of D. Przeworska-Rolewicz and to the approach of P. Bittner, in which two algebraic systems are considered - a ring of operators and a linear space, in Dimovski's scheme it is considered a single algebraic system - the ring of the multiplier quotients. This approach was described first in the Dr. Sc. thesis of Dimovski (see [66]) and then, in his monograph [67]. The advantage of using multipliers quotients instead of convolutional ones, as it is in Mukisinski's approach, can be seen in the operational calculi of functions of several variables (papers [71],[72],[65]). The more, there exist cases when the convolutional quotients' approach is not applicable (see [18],[19]), while the multipliers' one works successfully.

To summarize, Dimovski's basic contributions can be classified in the following domains of mathematical analysis: operational calculus, integral transforms, theory of multipliers of convolutional algebras, expansions in eigenfunctions and associated functions, explicit characterization of commutants and automorphisms in them, linear nonlocal boundary value problems for equations of mathematical physics.

The recent papers of Dimovski and his collaborators (for some of them, see Section III of List of Publications) show the continuation and further developments and applications of his approach and ideas.

Virginia Kiryakova, MANAGING EDITOR OF "FCAA'

LIST OF SOME BASIC PUBLICATIONS
of Prof. Ivan Dimovski

I. Scientific Articles
1. Popular Papers and Surveys

[1] Is the Mathematics a Science? In: *Plenar Talks and Contributions, Conf. 25th Anniversary of Shoumen Univ. "Ep. K. Preslavski"* (1996), 25-37 (In Bulgarian).

[2] Mathematics and Reality, *"Matematika"* (1971), **3** (In Bulgarian).

[3] The new notebooks on geometry for VII and VIII grades/years/. In: *"Mathematics and Math. Education. Talks of 2nd Spring Conf. UBM"* (1973), 15-28 (In Bulgarian).

[4] Measurements of Arhimedian values. In: *"Mathematics and Math. Education. Talks of 30th Spring Conf. UBM"* (2001), 86-94 (In Bulgarian).

[5] On the bases of opetational calculus. In: *"Mathematics and Math. Education. Talks of 2nd Spring Conf. UBM"* (1974); 103-112 (In Bulgarian).;

[6] Contemporary operational calculus. *Fiz.-matemat. spsanie*, **19**, 1 (1976), 54-11 (In Bulgarian).

[7] The convolutional calculus. In: *"Mathematics and Math. Education. Talks of 7th Spring Conf. UBM"*, (1978), 46-49 (In Bulgarian).

[8] Convolutions of right inverse operators and representation of their multipliers. *Compt. rend. Acad. bulg. Sci.*, **31**, No 11 (1978), 1377-1380.

[9] The convolutional method in operational calculus. In: *Generalized Functions and Operational Calculus. Proc. Varna Conf.'1975* (1979), 69-68.

[10] Convolutions and approximation.In: *Constructive Function Theory. Proc. Conf. Blagoevgrad'1977* (1980), 301-306.

2. Convolutions, Operational Calculi, Multipliers and Commutants Related to Differentiation Operator

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[12] Operational calculus for the general linear differential operator of the first order. *Compt. rend. Acad. bulg. Sci.*, **26**, No 12 (1973), 1579-1582.

[13] On an operational calculus for vector-valued functions. *Math. Balkanica*, **4** (1974), 129-135.

[14] Convolutions for the right-inverse operators of the general linear differential operator of the first order. *Serdica*, **2** (1976), 82-86.

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[19] Mean-periodic operational calculi. Singular cases (Dimovski I., Skrnik, K.). *Fractional Calculus and Appl. Analysis*, **4**, No 2 (2001), 237-243.

[20] Bernouli operational calculus (Dimovski, I., Grozdev, S.). *"Mathematics and Math. Education. Talks of 9th Spring Conf. UBM"*, (1980); 30-36 (In Bulgarian).

[21] Discrete operational calculi for two-sided sequences (Dimovski I., Kiryakova V.). In: *"Application of Fibonacci numbers"* (Eds. E. E. Bergum et al.), **5**, Kluwer (1992), 159-168.

[22] Generalizations of finite and discrete Fourier transforms. In: *"Proc. International Workshop on Recent Advances in Applied Mathematics (RAAM)"*, Kuwait (1996), 101-105.

[23] Representation of operators which commute with differentiation in an invariant hyperplane. *Compt. rend. Acad. bulg. Sci.*, **31**, No 10 (1978), 1245-1248.

[24] Convolutions, multipliers and commutants for the backward shift operator (Dimovski I., Mineff D.). *Pliska*, **4** (1981), 128-136.

[25] Automorphisms of C which commute with the integration operator (Dimovski I., Micheva S.). *Integral Transforms and Special Functions,* **4**, No 1-2, (1996), 69-76.

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3. Convolutions and Finite Integral Transforms for Second Order Linear Differential Operators

[31] Two new convolutions for linear right-inverse operators of d^2/dt^2. *Compt. Rend. Acad. bulg. Sci.*, **29** (1976), 25-28.

[32] Operational calculus for general linear differential operator of second order (Dimovski I.; Bozhinov N.). *Compt. Rend. Acad. bulg. Sci.*, **28**, No 6 (1975), 727-730.

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4. Convolutions and Integral Transforms, Related to the Hyper-Bessel Operator

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[43] On an operational calculus for a differential operator. *Compt. rend. Acad. bulg. Sci.*, **21**, No 2 (1968), 513-516.

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